false. This latter principle is 'unhealthy', because human beings need to hold fallible beliefs in order to function successfully. (In my [1975*b*], p. 311, I gave an example of a reason why someone might 'need to know' what the correct theory of Atlantic was; if necessary, one could construct another hypothetical situation with the same methodologically-problematic features but in which the motive for finding the true theory at one or both levels was much more tangible and pressing.) What people need from the philosophy of science is a set of principles showing how to choose beliefs which are relatively 'good' in some sense; scepticism of the Popperian kind is part of a methodology which, arguably, serves just this purpose, but the stronger scepticism entailed by d'Agostino's remarks conflicts with that need.

The principle that theories should be rejected whenever they are less than ideal from the methodological point of view, regardless of whether methodologically-preferable alternatives are available, is likely in practice to require the rejection of all theories ever constructed in any domain (Lakatos [1970]). D'Agostino perhaps means only to argue that rejection of all extant alternatives is appropriate just when the methodologically-superior alternative at one level is incompatible with the methodologically-superior alternative at another level, as I suggest may be characteristic for linguistics in particular. But if agnosticism is not in general justified by the fact that all extant theories in a given domain are in some way or other methodologically imperfect, what is it about the special kind of methodological difficulties in my example that makes agnosticism permissible here?

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AD HOCNESS AND THE APPRAISAL OF THEORIES *

1 THE BAYESIAN ANALYSIS OF AD HOCNESS.

In a recent note Schaffner ([1974]) has given a formal discussion of the notion of *ad hoc*ness in terms of a Bayesian model for the appraisal of theories. Schaffner develops his general ideas in the context of a critique of Zahar's [1973] which was concerned with the particular problem of comparing the Einstein and

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Lorentz research programmes. Zahar suggests the following analysis of *ad hoc*ness[1]:

> *Ad hoc*$_1$: A theory is said to be *ad hoc*$_1$ if it has no novel consequences as compared with its predecessor.
>
> *Ad hoc*$_2$: [A theory] . . . is *ad hoc*$_2$ if none of its novel predictions have been actually 'verified'.
>
> *Ad hoc*$_3$: [A] . . . theory is said to be *ad hoc*$_3$ if it is obtained from its predecessor through a modification of the auxiliary hypotheses which does not accord with the spirit of the heuristic of the programme.

Zahar explains the meaning of novelty as follows[2]

> A fact will be considered novel with respect to a given hypothesis if it did not belong to the problem-situation which governed the construction of the hypothesis.

Schaffner begins his elucidation by discussing the notions of *ad hoc*$_1$ and *ad hoc*$_3$. The first he describes as a logical dream, since the novel consequences of a theory cannot in practice be 'surveyed', so the question of whether a theory is *ad hoc*$_1$ can only be discussed relative to the extent to which novel consequences have been looked for at the particular epoch of the evaluation. *Ad hoc*$_3$ Schaffner claims to be 'vague to the point of inapplicability'. For Schaffner *ad hoc*$_2$ is 'close to the sense in which *ad hoc* is used in science' and he announces that his Bayesian analysis will be brought to bear on this sense of *ad hoc*.

Denoting by $p(T/b\ \&\ e)$ the probability of a theory T to be true in the light of background knowledge b and the positive outcome e of some experiment not part of b, by $p(T/b)$ the prior assessment of T, by $p(e/T\ \&\ b)$ the probability of obtaining e given T and b, and by $p(e/b)$ the probability of obtaining e on the basis of background knowledge alone, Schaffner writes

$$p(T/b\ \&\ e) = \frac{p(T/b)\cdot p(e/T\ \&\ b)}{p(e/b)} \tag{1}$$

If T explains e we can set $p(e/T\ \&\ b) = 1$ so in this case we obtain

$$p(T/b\ \&\ e) = p(T/b)/p(e/b) \tag{2}$$

Schaffner proceeds to discuss *ad hoc*ness as a property of an hypothesis as a constituent part of a theory, but in order to keep the argument as simple as possible we shall follow Zahar in considering the *ad hoc*ness of theories. Translated into these terms Schaffner's idea is that a theory T gives an *ad hoc* explanation of an experimental result e if $p(T/b)$ is close to zero and $p(e/b)$ is significantly larger than $p(T/b)$. The argument for $p(T/b)$ being small is that it has no 'theoretical support' (or indeed empirical support other than e itself). This looks suspiciously like a reference to *ad hoc*$_3$, and Schaffner now appears to be claiming that a theory is *ad hoc*$_2$ partly in virtue of its being *ad hoc*$_3$. So he is not really giving an independent analysis of *ad hoc*$_2$ at all.

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[1] Zahar [1973], p. 101. In his [1978] Zahar rephrases his definition in terms of a notion of empirical non-*ad hoc*ness expressed as a three-place relation between an observation statement, a theory and a heuristic. See also in this connection the very clear account of empirical support given by Worrall in his [1978]. However this more recent work has somewhat obscured the important distinctions drawn by Zahar in his [1973].

[2] Zahar [1973], p. 103.

The lack of novelty in the prediction of e is associated by Schaffner with a high value for $p(e/b)$. On Zahar's account this is a necessary condition for lack of novelty, but not a sufficient condition. We proceed to show how using Zahar's notion of novelty one can get an 'internal' Bayesian analysis of *ad hoc*$_2$.

We express Bayes's theorem in the following familiar way

$$p(T/b \,\&\, e) = \frac{p(T/b) \cdot p(e/T \,\&\, b)}{p(e/T\&b) \cdot p(T/b) + p(e/\sim T\&b) . p(\sim T/b)} \tag{3}$$

where $\sim T$ denotes the negation of T.

We follow Schaffner in interpreting $p(A/B)$ as the degree of belief that A is true given that B is true. Note also that e is supposed in equation (3) to refer to a prediction made by the theory T (we could perhaps write e_T to emphasise this important point) so $p(e/\sim T \,\&\, b)$ means the probability that the prediction e_T derived from the theory T is a true prediction given that the background information b is true but the theory T is false (*i.e.* its consequences are not guaranteed to be true although 'by accident' they may be true).

Writing $p(T/b) = x$, $p(e/\sim T \,\&\, b) = \epsilon$
and taking $p(e/T \,\&\, b) = 1$ and using $p(\sim T/b) = 1-x$

$$p(T/b \,\&\, e) = \frac{x}{x+\epsilon(1-x)} \tag{4}$$

We define an enhancement ratio γ by

$$\gamma = \frac{p(T/b \,\&\, e)}{p(T/b)}$$

whence using (4) we obtain the simple result

$$\gamma = \frac{1}{x+\epsilon(1-x)} \tag{5}$$

We can now explain that if a theory T is *ad hoc*$_2$ with respect to the experiment e then $\epsilon = 1$, *i.e.* the explanation of e by T in no way depends on the truth or falsehood of T, both of which eventualities lead with certainty to the result e. This is just what a scientist means when he says T was an *ad hoc* explanation of e, namely T was devised for the express purpose of explaining e, so the explanation of e is guaranteed independently of whether T is true or false. To show the consistency of our analysis if we put $\epsilon = 1$ in (5) we get $\gamma = 1$, so the posterior and prior probabilities of T are equal (there is no enhancement) and this again is just what we expect from an *ad hoc* explanation of e, namely e itself gives us no additional information for assessing the truth of T.[1]

Notice that $p(e/b) = x+\epsilon(1-x)$ is equal to unity if $\epsilon = 1$, but that $p(e/b) = 1$ is achieved for $x = 1$ whatever the value of ϵ, so the explication of novelty in terms of low $p(e/b)$ (Schaffner) and small ϵ (Zahar) are by no means equivalent.

On our analysis if $x \ll \epsilon \ll 1$, then $\gamma \simeq 1/\epsilon$, so in this case we get a big enhancement and the theory is far from being *ad hoc*. Noting that under these conditions $p(e/b) \simeq \epsilon$, we see that this is a situation in which Schaffner would claim that the theory was *ad hoc*, which highlights the way in which his analysis differs from ours. Effectively Schaffner requires γx to be small as his condition of *ad hoc*ness. He is thus concerned with the absolute value of the probability of a theory after it has explained some experimental finding. If this absolute value is still small the theory is to be regarded as an *ad hoc* explanation of the experiment. On our account the important aspect in assessing *ad hoc*ness for this case is not the absolute value of the probability but the enhancement ratio. Our point against Schaffner is not that his analysis may not explicate some legitimate sense of the ambiguous appellation *ad hoc*, but that it fails to explicate Zahar's very important notion of *ad hoc*$_2$. There is no inconsistency here on Schaffner's part, since he finds Zahar's account of *ad hoc*$_2$ inadequate in respect of the definition of novelty involved with its historical associations, but we would maintain that Zahar's sense of *ad hoc*$_2$ is the one that ought to be explicated since it is the one most importantly used in science.

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[1] What we have shown is that $\epsilon = 1$ is a necessary condition for T to be an *ad hoc*$_2$ explanation of e. To justify $\epsilon = 1$ as a sufficient condition we must invoke a principle of insufficient reason, *viz.*, if there is no reason for the community of scientists to entertain with non-vanishing prior probability only theories which explain e, then they will not so constrain their choice of alternative theories.

2 THE CASE OF MULTIPLE PREDICTIONS

To develop our analysis a little further we can consider how a theory builds up a favourable appraisal as it makes a number of successful predictions $e_1, e_2 \ldots e_n$ say. Denote by p_s the posterior probability $p(T/b \,\&\, e_1 \,\&\, e_2 \ldots \&\, e_s)$ after s successful predictions. Assuming for simplicity that successive appraisals are all associated with the same value of ϵ, we can clearly write

$$p_n = \gamma^{(n)} \cdot \gamma^{(n-1)} \ldots \gamma^{(1)} \cdot p_0$$

where $p_0 = x$ according to our previous notation

and

$$\gamma^{(s)} = \frac{1}{p_{s-1}(1-\epsilon)+\epsilon}$$

$$p_{s+1} = \gamma^{(s+1)} \cdot p_s$$

The solution of this recursion is by inspection or more simply by replacing ϵ by ϵ^n in the formula for p_1 (see (4) above). We obtain[1]

$$p_n = \frac{1}{1-\epsilon^n+\epsilon^n/x} \tag{6}$$

We can also ask what is the probability for the $(n+1)$th prediction e_{n+1} being correct if the theory has already made n successful predictions $e_1 \ldots e_n$. Denoting this probability by $p(e_{n+1})$ we clearly obtain the result

$$p(e_{n+1}) = \epsilon + \frac{1-\epsilon}{1-\epsilon^n+\epsilon^n/x} \tag{7}$$

If ϵ is a small quantity (*i.e.* $\ll 1$) which will be the case if T is non-*ad hoc*$_2$ with respect to all the predictions, we can write the following formulae which will be perfectly satisfactory for the subsequent discussion

$$p_n \simeq \frac{1}{1+\epsilon^n/x} \tag{8}$$

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[1] This result will be recognised as a special case of the general treatment given by Keynes in his [1921], pp. 235–6.

$$p(e_{n+1}) \simeq \epsilon + p_n \tag{9}$$

$$\gamma^{(n)} \simeq \frac{1}{\epsilon + p_{n-1}} \tag{10}$$

We note the following features

(1) The value of p_n depends entirely on the ratio ϵ^n/x.
For $\epsilon^n/x \gg 1$, $p_n \ll 1$ and for $\epsilon^n/x \ll 1$, $p_n \simeq 1$.
At the critical point $\epsilon^n = x$ we have $p_n \simeq 1/2$.
So if initially $x \ll \epsilon$ as n increases p_n will rise steeply as we reach the value $n = \ln x/\ln \varepsilon$.

(2) So long as $p_n \ll \epsilon$ we have $p(e_{n+1}) \simeq \epsilon$, but as p_n builds up towards unity so does $p(e_{n+1})$.

(3) So long as $p_{n-1} \ll \epsilon$, $\gamma^{(n)} \simeq 1/\epsilon$, but as p_{n-1} builds up towards unity the enhancement factor $\gamma^{(n)}$ also tends to unity.

To take a concrete case we illustrate in the accompanying figure p_n and $p(e_{n+1})$ as functions of n for the particular choice $x = 0{\cdot}01$, $\epsilon = 0{\cdot}1$.

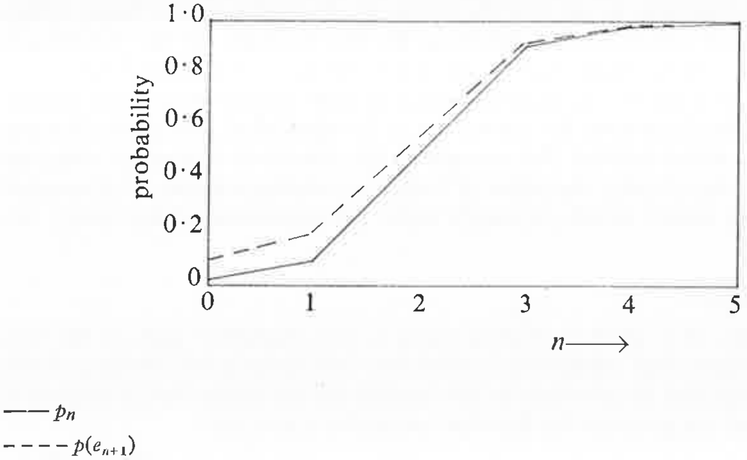

Figure 1.

3 QUANTITATIVE PREDICTIONS

We can apply this analysis to the important case of quantitative predictions.[1] Suppose a theory T predicts correctly an experimental result which is known to an accuracy of n significant figures. We assume as part of our background knowledge that the order of magnitude of the result is known, *i.e.* we disregard the prediction of zeros occurring before or after the significant figures in the experimental result. As a concrete illustration we cite the theoretical predictions by quantum electrodynamics of the anomaly in the hydrogen spectrum known as the Lamb shift and the anomaly in the magnetic moment of the electron. For example the latter is now known experimentally to be $(11596524 \pm 2) \times 10^{-10}$ Bohr magnetons[1] whereas the theoretical prediction is $(11596524 \pm 6) \times 10^{-10}$ Bohr magnetons.[2] We thus have remarkable agreement to seven significant figures. Clearly if we regard the prediction of each significant figure as an independent event then the appropriate value to take for ϵ is 0·1 since a false theory would have ten equal possibilities for filling in each digit. The question of what value to take for x is somewhat arbitrary. In agreement with Schaffner we do not follow a purely logical approach and set $x = 0$. For our purpose x reflects the scientist's confidence in the new theory T. One could argue that a scientist would not spend great efforts developing the consequences of a theory he did not believe in. By analogy with the situation in the Bayesian analysis of significance testing (see for example Redhead ([1974]) we could take $x = \frac{1}{2}$. Perhaps more realistically we should take x around 0·01 and adopt the sociological rule that unless a scientist has a one per cent level of confidence in the truth of his theory he would not seriously investigate it.[3] With this choice of parameters we see that the build-up of confidence in a theory which makes correct quantitative predictions, as the accuracy of the experiment increases, would be illustrated by the graph for p_n in the figure.[4] Of course $p(e_{n+1})$ is only given by our analysis so long as other factors which could potentially influence the results are known not to be significant. For a certain value of n this condition will fail. For example in the case of the anomalous magnetic moment of the electron the effect of hadronic couplings introduce theoretical uncertainties which would ultimately make the prediction of the theory unreliable.[5]

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[1] The successful detailed quantitative predictions of a theory in respect of phenomena quite different from those which the theory was originally proposed to deal with have always attracted the attention of scientists. To take an example at random, in referring to his early work on the ground state of helium Hylleraas comments in his [1963] (p. 42) 'The end result of my calculation was . . . greatly admired and thought of as almost a proof of the validity of wave mechanics . . . in the strict numerical sense'.

CONCLUSION

The concept of a novel prediction plays a very important part in the way scientists assess their confidence in theories. Our analysis has shown how the intuitive appraisal by scientists in this respect can be understood if degrees of commitment are governed by Bayesian rationality constraints.

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[1] Van Dyck *et al.* [1977].

[2] See value quoted in Calmet *et al.* [1977]. For a good account of the fluctuating agreement between theory and experiment the reviews by Lautrup, Peterman and de Raphael [1972] or Rich and Wesley [1972] may be consulted.

[3] We may refer to Shimony's concept of commitment to a theory (see his [1970], pp. 94–5). The degree of commitment measures the scientist's belief that the theory T belongs to the equivalence class of all theories which give the same true observational predictions within the domain of current experimentation and that the 'true' theory 'generalises' in some sense the concepts embodied in T. Commitment measures our belief, not that a theory is true, but that it points the way to the truth.

[4] It is easy to see that our confidence in a theory at a given level of accuracy for agreement between theory and experiment does not depend on the particular scale of notation used to express the result.

[5] According to Rich and Wesley ([1972]) the known hadronic contribution to the electron anomaly would affect the tenth significant figure.

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